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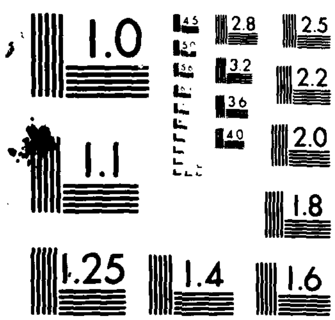
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THE CANCELLATION RATIO OF SCATTERED INTERFERENCE

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THE CANCELLATION RATIO OF SCATTERED INTERFERENCE

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I. INTRODUCTION

A method for automatically canceling jamming signals scattered into the main beam of a radar or communications antenna has been described, developed, and analyzed in three previous reports [1,2,3]. The cancellation technique was developed in the first two reports [1,2] for the case of a single jammer and when the jammer and scatterers are moving with only moderate velocities relative to the radar. In the third report [3], this technique was extended, again for the single jammer case, to the cancellation of scattered interference in a radar moving with a substantial velocity with respect to the scatterers and jammers.

In the first two reports [1,2], the development of the generalized sidelobe canceler (SLC) for canceling scattered interference was based on the following assumptions:

1. The main beam of the radar or communications antenna is high gain compared with the auxiliary antennas (AUXs) used for cancellation. As a consequence, the low gain AUXs receive normal radar or communications signals only imperceptibly compared with the normal main beam receiver.
2. The normal radar or communications signals and the interference are uncorrelated.
3. The direct jamming signal is received both by an AUX and by the main beam, but in the direction of the sidelobes of the main beam.

4. The jamming energy scattered by the scatterers and received in the main beam is received only insignificantly by an AUX.

5. The scatterers and jammers move only with moderate velocities relative to the radar.

In the third report [3], the fifth assumption was modified to include a radar or communications system moving rapidly with respect to the jammers and scatterers, i.e.,

5'. The scatterers and jammers move slowly relative to one another. The radar or communications system moves with a substantial velocity with respect to the scatterers and jammers.

With assumption 5', an optimum adaptive criterion was found in the third report [3] for canceling jamming scattered into the main beam of a radar or a moving vehicle. This criterion will be developed and analyzed further during the next reporting period. In this report, attention will be restricted to the adaptive criterion developed in the first two reports [1,2] under assumption 5.

Using assumptions 1 through 5, a recursive algorithm of the type discussed by Widrow and Howells-Applebaum was developed for the cancellation of both direct and scattered jamming signals [1]. Also, an analysis was made of the convergence rate of this algorithm [1]. It was demonstrated that the convergence time for this generalized SLC is strongly dependent on both the tap spacing and the receiver pass band.

In the second report [2], again using assumptions 1-5, a method was devised for computing precisely the expected noise residue remaining after

cancellation of the scattered jamming. This information was used to find a figure of merit, called the cancellation ratio (C.R.), for scattered jamming. The C.R. measures how well a generalized sidelobe canceler can be expected, on the average, to cancel the jamming scattered into the main beam of a radar or communications system. The C.R. for scattered jamming was found specifically for the case of a band-limited receiver with a flat spectral shape [2].

In this report, the previously developed method [2] will be used to find the C.R. for scattered jamming in a non-band-limited case. Specifically, the pass band of the receiver is assumed to be due to a single pole network. Also, the results of an actual computation of the C.R. will be given for the band-limited receiver case developed in Ref. 2.

II. NOISE PROCESSES OF ADAPTIVE CANCELER OF SCATTERED JAMMING

Following Ref. 3, the problem of finding the noise residue after the cancellation of scattered interference will be summarized. The problem will then be specialized in order to find the C.R. of scattered jamming in a certain non-band-limited case. The C.R. for the band-limited case was found previously [3].

Figure 1 is provided again to show the geometry of the scattered interference problem. Under assumption 4 of the previous section, the auxiliary antenna receives

$$x_R\left(t\right) = J\left(t - R_{d/c}\right) \quad . \tag{2.1}$$

Let a discrete set of randomly distributed scatterers, e.g., chaff, lie only in the range interval $\left(R_o - D/2,\ R_o + D/2\right)$. Then the main beam of the radar receives the scattered chaff process,

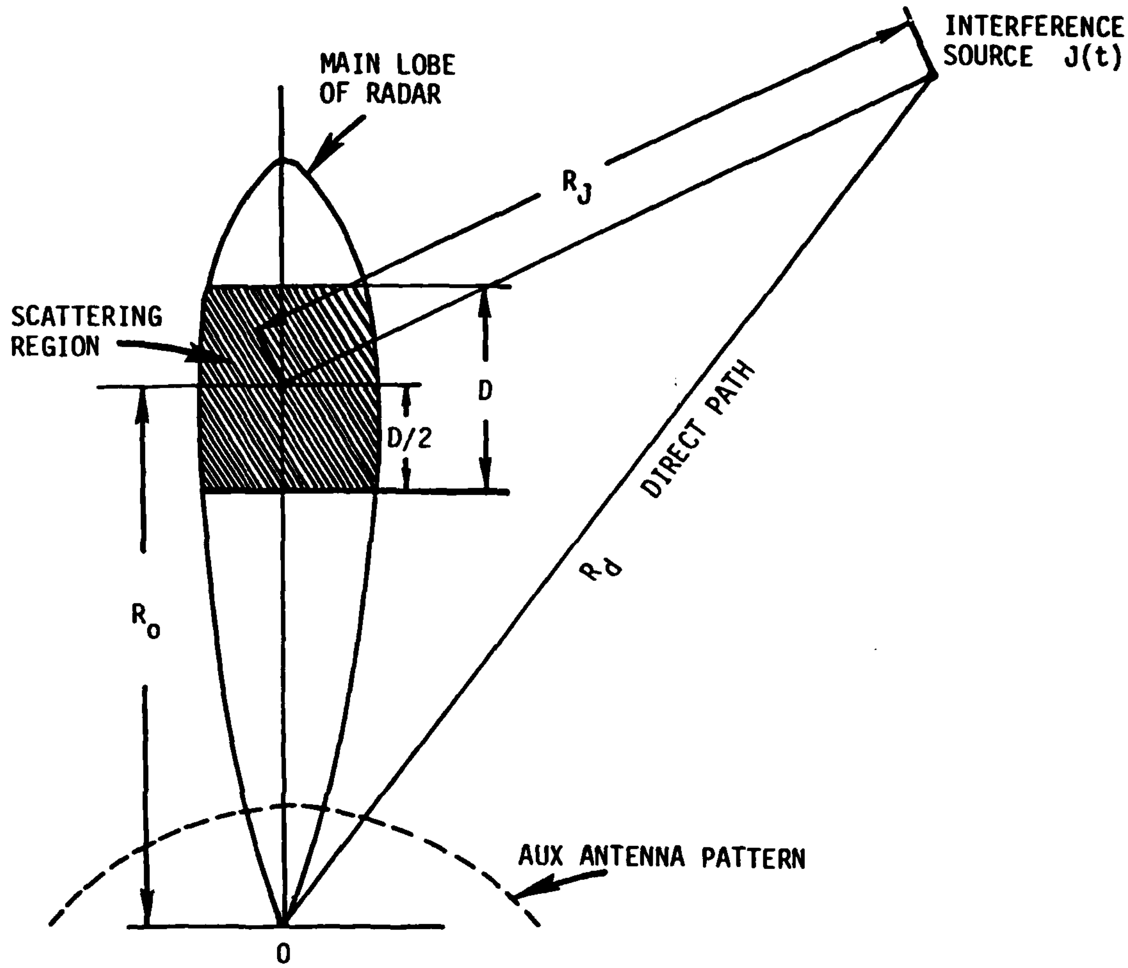

Fig. 1 — Geometry of scattered interference

$$y(t) = \sum_{T_- \leq T_k \leq T_+} \alpha_k J(t - T_k) , \tag{2.2}$$

where

$$T_- = (R_o + R_J - D/2)/c ,$$
$$T_+ = (R_o + R_J + D/2)/c , \tag{2.3}$$

α_k = the scattering coefficient for the k-th scatterer, and

T_k = the one-way delay time from scatterer to radar.

In both Eq. (2.1) and Eq. (2.2), the receiver noise is neglected; also, in Eq. (2.2), the direct jamming signal in the radar sidelobes is assumed to be zero (see Ref. 2 for further discussion).

The adaptive canceler for scattered jamming uses the circuit in Fig. 2 to form an "error" signal $z(t)$ between $y_R(t)$ and the delayed and weighted versions of $x_R(t)$. The weights w_k are chosen to minimize $E|z(t)|^2$ over some interval. After this minimization, $z(t)$ is the radar signal with the scattered jamming removed or canceled.

It is convenient for purposes of analysis to express $y(t)$ in Eq. (2.2) directly in terms of a certain delayed version of the received AUX signal $x_R(t)$ in Eq. (2.1). Toward this end, first define in Eq. (2.3)

$$R_1 = R_o + R_J , \tag{2.4}$$

and note, from Fig. 1, that

$$R_d \leq R_1 . \tag{2.5}$$

In terms of R_1 in Eq. (2.4), $y(t)$ in Eqs. (2.2) and (2.3) can be expressed

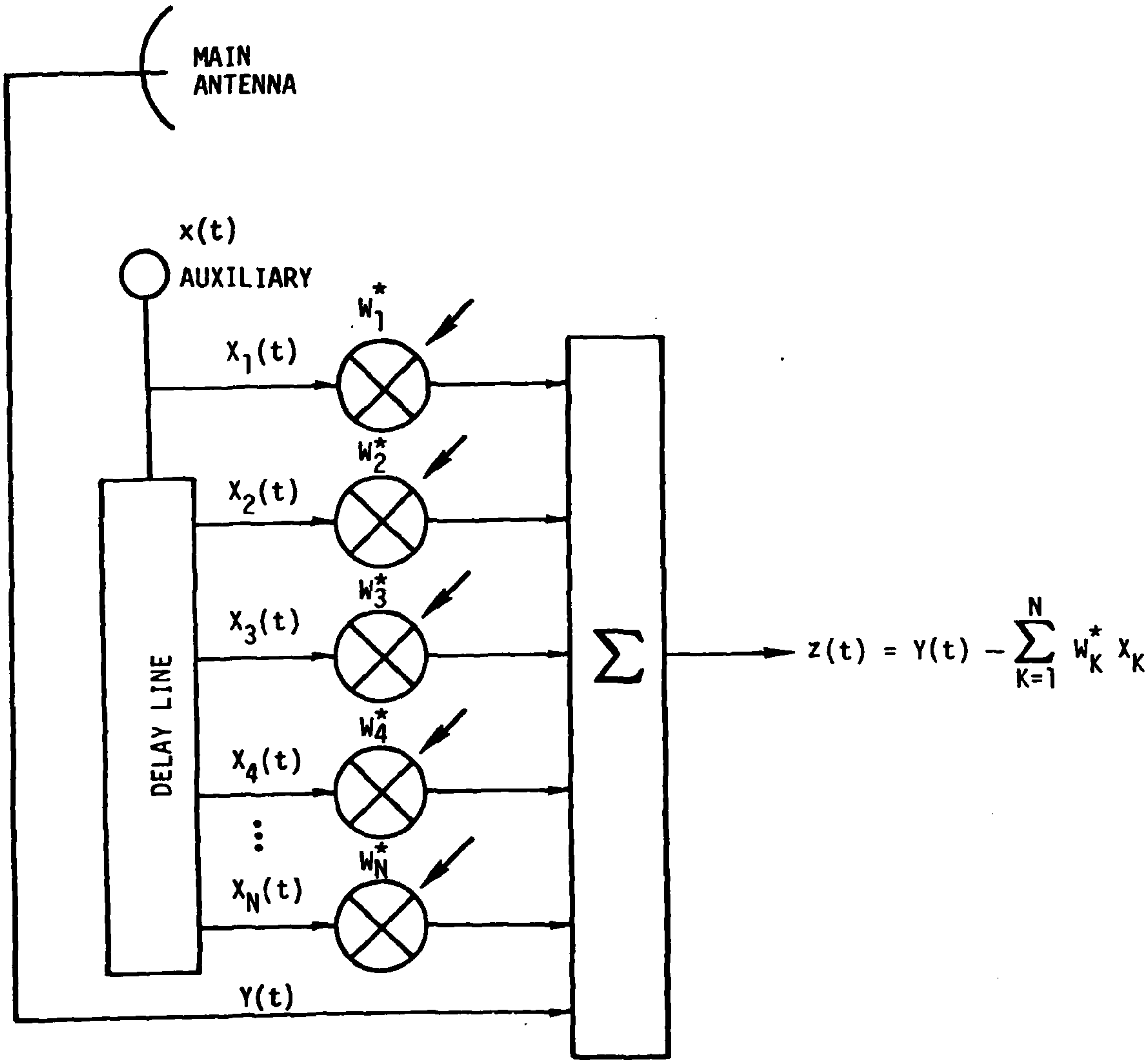

Fig. 2 — Cancellation of scattered interference

evidently as

$$y(t) = \sum_{-D/2c \le T_k - R_1/c \le D/2c} \alpha_k J(t - T_k)$$

$$= \sum_{-D/2c \le T_k - R_1/c \le D/2c} \alpha_k J\left(t - \left[T_k - R_1/c\right] - R_1/c\right) \qquad (2.6)$$

$$= \sum_{-D/2c \le \tau_k \le D/2c} \alpha_k J(t - \tau_k - R_1/c),$$

where

$$\tau_k = T_k - R_1/c \quad .$$

By Eq. (2.5), $\left[R_1 - R_d\right]/c$ is a positive time delay. Suppose now that signal $x_R(t)$ in Eq. (2.1) is delayed by this time delay to yield

$$\begin{aligned} x(t) &= x_R\left(t - \left[R_1 - R_d\right]/c\right) \\ &= J\left(t - \left[R_1 - R_d\right]/c - R_d/c\right) \qquad (2.7) \\ &= J(t - R_1/c) \end{aligned}$$

as the AUX signal. In terms of $x(t)$ in Eq. (2.7), $y(t)$ in Eq. (2.6) is expressible as

$$y(t) = \sum_{-D/2c \le \tau_k \le D/2c} \alpha_k x(t - \tau_k) \quad . \qquad (2.8)$$

This form for $y(t)$ is useful for computing the jamming residue left after cancellation.

As in Ref. 2, the sum for $y(t)$ can be expressed in the even more useful form as a Reimann-Stieltjes stochastic integral,

$$y(t) = \int_{-D/2c}^{D/2c} x(t - \tau) \; d\,\zeta(\tau), \tag{2.9}$$

where $\zeta(t)$ is a stochastic orthogonal-increment process of Poisson events. Specifically, the sequence τ_1, τ_2, τ_3, ..., of scattering times in Eq. (2.8) are assumed to occur as a Poisson process of rate, λ events per second. Associated with this sequence of events are the complex amplitudes, α_1, α_2, α_3, ..., of the scattering coefficients of the scatterers. Since the orientations of different scatterers are unrelated, it can be assumed that α_1, α_2, α_3, ..., constitute a sequence of mutually independent, identically distributed, random variables of zero mean.

In terms of the sequence of the "event" 2-tuples, (τ_1, α_1), (τ_2, α_2), (τ_3, α_3), ..., $\zeta(t)$ in Eq. (2.9) is a stochastic process with sample functions which are constant between events at times τ_k and τ_{k+1} and change by the amplitude α_k at the event time τ_k. That is, $\zeta(t)$ is constant for $\tau_k \leq t < \tau_{k+1}$, and

$$\zeta(t) = \zeta(\tau_k -) + \alpha_k \quad \text{for} \quad (k = 1, 2, \ldots), \tag{2.10}$$

where $\zeta(\tau_k -)$ is the value of $\zeta(t)$ immediately prior to event at time τ_k.

One property of process $\zeta(t)$ is found to be

$$E\left[\zeta(t_2) - \zeta(t_1)\right]^* \left[\zeta(t_4) - \zeta(t_3)\right] = 0 \quad ,$$

if intervals (t_1, t_2) and (t_3, t_4) have no points in common; this is the orthogonal increment property. If intervals (t_1, t_2) and (t_3, t_4) reduce to infinitesimal intervals of form $(t, t + dt)$, then the orthogonal increment property becomes

$$E\left[d\,\zeta^*\left(t'\right)\,d\,\zeta\left(t\right)\right] = \lambda\,\beta\,\delta\left(t - t'\right)\,dt\,dt' \quad ,$$

where

$$d\,\zeta\left(t\right) = \zeta\left(t + dt\right) - \zeta\left(t\right) \,. \tag{2.11}$$

Also, in Eq. (2.10), $\delta\left(t - t'\right)$ is the Dirac delta function,

$$\beta = E\left|\alpha_k\right|^2 \quad \text{for} \quad \left(k = 1, 2, 3, \ldots\right) \tag{2.12}$$

and

$$\lambda = \rho\,c \quad , \tag{2.13}$$

where ρ is the number of scatterers observed by the radar per unit distance in range and c is the velocity of light.

In order to estimate the degree to which scattered jamming can be canceled with the finite tapped delay line in Fig. 2, it is helpful to utilize the spectral representation of $x\left(t\right)$, the jamming signal. The spectral representation of $x\left(t\right)$ has the form (see Ref. 2 for more discussion) of a Riemann-Stieltjes Fourier transform,

$$x\left(t\right) = \int_{-\infty}^{\infty} e^{2\pi i f t}\,H\left(f\right)\,d\,B\left(f\right) \,, \tag{2.14}$$

where $H\left(f\right)$ is the frequency response function that characterizes $x\left(t\right)$, and $B\left(f\right)$ is the complex Brownian noise process with the orthogonal increment property,

$$E\left\{d\,B^*\left(f\right)\,d\,B\left(t'\right)\right\} = N_0\,\delta\left(f - f'\right)\,df\,df' \quad . \tag{2.15}$$

Here, N_0 is the two-sided spectral noise density of the underlying white noise process.

The stochastic integral for $y(t)$ in Eq. (2.9) and the spectral representation of $x(t)$ in Eq. (2.14) will be used to estimate the residue of scattered jamming after cancellation. Previously [2], it was assumed that $H(f)$ in Eq. (2.14) was band-limited. In the present analysis, it is assumed that $H(f)$ is non-band-limited. For the present case, the delay time Δ between taps can be arbitrary.

III. CANCELLATION RATIO OF SCATTERED JAMMING IN A NON-BAND-LIMITED CASE

Suppose that, at the receiver base band, the jammer noise process can be approximated by white noise that has been passed through a single pole network. The time response function of such a network has form

$$\begin{aligned} K(t) &= e^{-\alpha t} \quad \text{for} \quad t \geq 0 \\ &= 0 \quad \text{for} \quad t < 0 \ , \end{aligned}$$

where α is the reciprocal of the time constant, e.g., in a resistance (R) and capacitance (C) network, $\alpha = 1/RC$. The frequency response function $H(f)$ is the Fourier transform of $K(t)$. That is,

$$H(f) = \int_{-\infty}^{\infty} e^{-i w t} K(t)\, dt = 1/(\alpha + i w) \ , \tag{3.1}$$

where $w = 2\pi f$, and f is the frequency of the so-called system function of the network. Substituting Eq. (3.1) into Eq. (2.14) yields,

$$x(t) = \int_{-\infty}^{\infty} e^{i w t} \frac{1}{\alpha + i w}\, d B(f) \ , \tag{3.2}$$

where $w = 2\pi f$, and

$$E\left\{d B^{*}(f)\, d B(f')\right\} = N_0\, \delta(f - f')\, df\, df' \ , \tag{3.3}$$

as the spectral representative of the jammer that is received by the AUX.

The autocovariance function of $x(t)$ in Eqs. (3.2) and (3.3) is

$$R_x(t - t') = E\, x^*(t')\, x(t)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{e^{-i\,w\,t'}\, e^{i\,w'\,t}}{(\alpha - i\,w)(\alpha + i\,w')} E\, d\,B^*(f)\, d\,B(f')$$

$$= \frac{N_0}{2\pi}\int_{-\infty}^{\infty} \frac{e^{i\,w(t - t')}}{\alpha^2 + w^2}\, d\,w \quad , \tag{3.4}$$

since $w = 2\pi f$. For $\tau = t - t' > 0$, the integral in Eq. (3.4) is easily seen to be a contour integral along the x-axis that loops clockwise around the upper half plane. The pole of the integrand within this contour is $i\,\alpha$, so that, by the Cauchy residue theorem,

$$R_x(\tau) = \frac{N_0}{2\alpha}\, e^{-\alpha\tau} \quad \text{for } \tau > 0 \quad .$$

If $\tau = t - t' < 0$, a similar argument yields,

$$R_x(\tau) = \frac{N_0}{2\alpha}\, e^{\alpha\tau} \quad \text{for } \tau < 0 \quad .$$

Combining these results obtains

$$R_x(\tau) = \frac{N_0}{2\alpha}\, e^{-\alpha|\tau|} \tag{3.5}$$

as the autocovariance function of the $x(t)$ process.

The purpose of the generalized sidelobe canceler (SLC) circuit, shown in Fig. 2, is to create a "new beam" $z(t)$ by subtracting from the "old beam" $y(t)$ a sum of a weighted and delayed version of the AUX signal. Using $x(t)$ in Eq. (3.2), a delayed version of the AUX signal, this new beam has the form

$$z(t) = y(t) - \sum_{k=-N}^{N} W_k^* \, x(t - k\Delta) \quad , \tag{3.6}$$

where Δ is the sampling interval and W_k for $(k = -N, \ldots, -1, 0, 1, \ldots, N)$ are 2 N + 1 complex sample weights.

Define the row vectors

$$W = \left(W_{-N}, \ldots, W_{-1}, W_0, W_1, \ldots, W_N\right)^T \text{ and}$$
$$X(t) = \left(x(t - N\Delta), \ldots, x(t - \Delta), x(t), x(t + \Delta), \ldots, x(t + N\Delta)\right)^T \quad , \tag{3.7}$$

where "T" denotes transpose. Then, $z(t)$ in Eq. (3.6) can be expressed in form

$$z(t) = y(t) - W^* X(t) \quad , \tag{3.8}$$

where "*" denotes conjugate transpose, in terms of the $(2N + 1)$-element column vectors W and $X(t)$ in Eq. (3.7).

Again following Ref. 2 (Section IV), let E_x denote the conditional expectation operator that operates <u>only</u> on the processes $x(t - k\Delta)$; the event process $\zeta(t)$ in Eq. (2.10) is held fixed under operator E_x. That is, if $U\left(x(t), \zeta(t)\right)$ is the stochastic function of both processes $x(t)$ in Eq. (3.2) and $\zeta(t)$ in Eq. (2.10), then

$$E_x\left\{U\left(x(t), \zeta(t)\right)\right\} = E\left\{U\left(x(t), \zeta(t)\middle|\zeta(t)\right)\right\}, \tag{3.9}$$

the conditional expectation of U, holding $\zeta(t)$ fixed.

The magnitude squared of $z(t)$ in Eq. (3.8) is given by

$$\left|z(t)\right|^2 = \left|y(t)\right|^2 - W^* X(t)\, y^*(t) - y(t)\, X^*(t)\, W$$
$$+ W^*\left[X(t)\, X^*(t)\right] W \quad .$$

Taking the conditional expectation with operator E_x yields

$$E_x\left|Z(t)\right|^2 = E_x\left|y\right|^2 - W^*\left(E_x\, X\, y^*\right) - \left(E_x\, X^*\, y\right) W + W^*\left(E_x\, X\, X^*\right) W \tag{3.10}$$

as the residual scattered jammer power after cancellation, where, for simplicity, the dependence on time t has been deleted.

If one takes differentials of the residual scattered jammer power $E_x\left|Z(t)\right|^2$ in Eq. (3.10) and sets the result equal to zero, the equation

$$- E_x\left(X\, y^*\right) + M\, W = 0 \tag{3.11}$$

for a stationary value in W, where

$$M = E_x\, X\, X^* \tag{3.12}$$

is the conditional covariance matrix of X. The solution of Eq. (3.11),

$$W_{opt} = M^{-1}\, E_x\left(X\, y^*\right) \tag{3.13}$$

yields the minimum value of the residual scattered jammer power $E_x\left|Z(t)\right|^2$, where M is given by Eq. (3.12).

A substitution of Eq. (3.13) into Eq. (3.10) yields

$$R = \underset{W}{\mathrm{Min}}\; E_x\left|Z(t)\right|^2 = E_x\left|y\right|^2 - Q\;, \tag{3.14}$$

where

$$Q = \left(E_x\, X\, y^*\right)^* M^{-1}\left(E_x\, X\, y^*\right) \tag{3.15}$$

as the minimum residual scattered jamming power that remains after cancellation with the optimum weight vector W_{opt} in Eq. (3.13). It remains to compute

R in Eq. (3.14) for the non-band-limited case where $x(t)$ has spectral representation in Eq. (3.2)

Using Eq. (2.9), the first term of Eq. (3.14) is

$$
\begin{aligned}
E_x|y|^2 &= \int_{-T}^{T} \int_{-T}^{T} E_x\, x^*(t-\tau)\, x(t-\tau')\, d\zeta^*(\tau)\, d\zeta(\tau') \\
&= \int_{-T}^{T} \int_{-T}^{T} R_x(\tau-\tau')\, d\zeta^*(\tau)\, d\zeta(\tau) ,
\end{aligned}
\tag{3.16}
$$

where

$$T = D/2c . \tag{3.17}$$

A substitution of Eq. (3.5) into Eq. (3.17) yields

$$E_x|y|^2 = \sigma^2 \int_{-T}^{T} \int_{-T}^{T} e^{-\alpha|\tau-\tau'|}\, d\zeta^*(\tau)\, d\zeta(\tau') , \tag{3.18}$$

where T is given by Eq. (3.17), and

$$\sigma^2 = N_0/2\alpha. \tag{3.19}$$

By Eqs. (3.12) and (3.5), the moment matrix M is given by

$$
\begin{aligned}
M = E_x X X^* &= \left(E\, x(t-k\Delta)\, x^*(t-\ell\Delta)\right) \\
&= \left(R_x\left([\ell-k]\Delta\right)\right) \\
&= \frac{N_0}{2\alpha}\left(e^{-\alpha|\ell-k|\Delta}\right) ,
\end{aligned}
\tag{3.20}
$$

where $(a_{\ell,k})$ denotes the matrix of the array $a_{\ell,k}$ of $(2N+1) \times (2N+1)$ elements. If, in Eq. (3.20), one lets

$$\rho = e^{-\alpha\Delta} \text{ and } \sigma^2 = N_o/2\alpha \ ,$$

then

$$M = \sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \dots \rho^{2N} \\ \rho & 1 & \rho & \dots \rho^{2N-1} \\ \rho^2 & \rho & 1 & \dots \rho^{2N-2} \\ \vdots & \vdots & \vdots & \vdots \\ \rho^{2N} & \rho^{2N-1} & \rho^{2N-2} & \dots 1 \end{bmatrix} \tag{3.21}$$

is an explicit expression for the covariance matrix of X as defined in Eq. (3.12). It is well known that matrix M in Eq. (3.21) has an explicit inverse given by

$$M^{-1} = \frac{1}{\sigma^2\left(1-\rho^2\right)} \begin{bmatrix} 1 & , -\rho & 0 & , \dots & 0 \\ -\rho & , 1+\rho^2, & -\rho & , \dots & 0 \\ 0 & , -\rho & , 1+\rho^2, & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & , & , 0 & , \dots & 1 \end{bmatrix} \tag{3.22}$$

where

$$\rho = e^{-\alpha\Delta} \text{ and } \sigma^2 = N_o/2\alpha \ . \tag{3.23}$$

The expression needed to compute R in Eq. (3.14) is, by Eqs. (2.9) and (3.5), the $\left(2\ N + 1\right)$-vector,

$$\begin{aligned} E_X\, X\, y^* &= \left(\int_{-T}^{T} E_X\, x\left(t - n\Delta\right)\ x^*\left(t - \tau\right)\ d\ \zeta^*(\tau) \right) \\ &= \sigma^2 \left(\int_{-T}^{T} e^{-\alpha|\tau - n\Delta|}\ d\ \zeta^*(\tau) \right) \\ &= \sigma^2 \left(\phi_n\right) , \end{aligned} \tag{3.24}$$

where (ϕ_n) denotes the column vector of the 2 N + 1 random variables,

$$\phi_n = \int_{-T}^{T} e^{-\alpha|\tau - n\Delta|} \, d\,\zeta^*(\tau) \,. \tag{3.25}$$

To compute Q in Eq. (3.15), first find, by Eqs. (3.22) and (3.24),

$$M^{-1} E_x X y^* = \frac{\sigma^2}{\sigma^2(1-\rho^2)} \begin{bmatrix} \phi_{-N} & -\rho\phi_{-N+1} & \\ -\rho\phi_{-N} & +(1+\rho^2)\phi_{-N+1} & -\rho\phi_{-N+2} \\ -\rho\phi_{-N+1} & +(1+\rho^2)\phi_{-N+2} & -\rho\phi_{-N+3} \\ \vdots & \vdots & \vdots \\ -\rho\phi_{N-1} & +\phi_N & \end{bmatrix} \tag{3.26}$$

$$\equiv \frac{1}{1-\rho^2}(\beta_n) \,,$$

where (β_n) denotes the column vector of the 2 N + 1 random variables given by

$$\begin{aligned} \beta_{-N} &= \phi_{-N} - \rho\phi_{-N+1} \\ \beta_{-N+k} &= -\rho\phi_{-N+k-1} + (1+\rho^2)\phi_{-N+k} - \rho\phi_{-N+k+1} \quad \text{for } (1 \le k \le 2N-1) \\ \beta_N &= -\rho\phi_{N+1} + \phi_N \,. \end{aligned} \tag{3.27}$$

In terms of the vectors in Eqs. (3.24) and (3.27), Q in Eq. (3.14) is given by

$$Q = \frac{\sigma^2}{1-\rho^2} \left[\phi^*_{-N}, \phi^*_{-N+1}, \ldots, \phi^*_N\right] \begin{bmatrix} \beta_{-N} \\ \beta_{-N+1} \\ \vdots \\ \beta_N \end{bmatrix}$$

$$= \frac{\sigma^2}{1-\rho^2} \sum_{n=-N}^{N} \phi^*_n \beta_n \,, \tag{3.28}$$

where random variables ϕ_n and β_n are given by Eqs. (3.24) and (3.27), respectively.

A substitution of Eqs. (3.24) and (3.27) into Eq. (3.28) yields

$$Q = \frac{\sigma^2}{1-\rho^2} \int_{-T}^{T} \int_{-T}^{T} \left[e^{-\alpha|\tau + N\Delta|} - \rho\, e^{-\alpha|\tau + (N-1)\Delta|} \right]$$

$$\cdot\, e^{-\alpha|\tau' + N\Delta|} d\,\zeta^*(\tau)\, d\,\zeta(\tau') + \sum_{n=-N+1}^{N-1} \int_{-T}^{T} \int_{-T}^{T}$$

$$\left[-\rho\, e^{-\alpha|\tau - (n-1)\Delta|} + (1+\rho^2)\, e^{-\alpha|\tau - n\Delta|} - \rho\, e^{-\alpha|\tau - (n+1)\Delta|} \right]$$

$$\cdot\, e^{-\alpha|\tau' - n\Delta|} d\,\zeta^*(\tau)\, d\,\zeta(\tau') + \int_{-T}^{T} \int_{-T}^{T} \left[-\rho\, e^{-\alpha|\tau - (N-1)\Delta|} \right.$$

$$\left. + e^{-\alpha|\tau - N\Delta|} \right] e^{-\alpha|\tau' - N\Delta|}\, d\,\zeta^*(\tau)\, d\,\zeta(\tau') \tag{3.29}$$

as an explicit expression for Q in terms of stochastic integrals. Thus, the residual scattered jamming power R that remains after cancellation is given by

$$R = E_x|y|^2 - Q \quad ,$$

where $E_x|y|^2$ and Q are given in Eqs. (3.18) and (3.29), respectively.

The "averaged" cancellation ratio over all ensembles of scatterers in the range interval $(R_o - D/2,\ R_o + D/2)$, shown in Fig. 1, is given by

$$\text{C.R.} = \frac{E(R)}{E\left(E_x|y|^2\right)} = \frac{E(R)}{E|y|^2} \quad . \tag{3.30}$$

The numerator of the cancellation ratio (C.R.) is, by taking the expected

value of R in Eq. (3.14),

$$E\,R = 2\lambda\beta\sigma^2 T\left[1 - \frac{1}{1-\rho^2}\left\{\frac{1}{2T}\int_{-T}^{T}\left[e^{-\alpha|\tau+N\Delta|} - \rho\, e^{-\alpha|\tau+(N-1)\Delta|}\right]e^{-\alpha|\tau+N\Delta|}\,d\tau + \sum_{n=-N+1}^{N-1}\frac{1}{2T}\int_{-T}^{T}\left[-\rho\, e^{-\alpha|\tau-(n-1)\Delta|} + (1+\rho^2)\,e^{-\alpha|\tau-n\Delta|} - \rho\, e^{-\alpha|\tau-(n+1)\Delta|}\right]\cdot e^{-\alpha|\tau-n\Delta|}\,d\tau + \frac{1}{2T}\int_{-T}^{T}\left[-\rho\, e^{-\alpha|\tau-(n-1)\Delta|} + e^{-\alpha|\tau-N\Delta|}\right]e^{-\alpha|\tau-N\Delta|}\,d\tau\right\}\right]. \tag{3.31}$$

The denominator of the cancellation ratio is, by Eq. (3.18),

$$E|y|^2 = 2\lambda\beta\sigma^2 T\ . \tag{3.32}$$

Substituting Eqs. (3.31) and (3.32) into Eq. (3.30) yields

$$C.R. = 1 - \frac{1}{2(1-\rho^2)T}\left\{\int_{-T}^{T}\left[e^{-\alpha|\tau+N\Delta|} - \rho\, e^{-\alpha|\tau+(N-1)\Delta|}\right]e^{-\alpha|\tau+N\Delta|}\,d\tau + \sum_{n=-N+1}^{N-1}\int_{-T}^{T}\left[-\rho\, e^{-\alpha|\tau-(n-1)\Delta|} + (1+\rho^2)\,e^{-\alpha|\tau-n\Delta|} - \rho\, e^{-\alpha|\tau-(n+1)\Delta|}\right]\cdot e^{-\alpha|\tau-n\Delta|}\,d\tau + \int_{-T}^{T}\left[-\rho\, e^{-\alpha|\tau-(n-1)\Delta|} + e^{-\alpha|\tau-N\Delta|}\right]e^{-\alpha|\tau-N\Delta|}\,d\tau\right\} \tag{3.33}$$

as the final expression for the cancellation ratio of scattered jamming in a certain band-limited case.

Computed results for the C.R., given in Eq. (3.33), will be given in a later report. In the next section, the cancellation ratio will be computed explicitly for the band-limited case studied previously [2].

IV. CANCELLATION RATIO WITH BAND-LIMITED INTERFERENCE

In the second quarterly progress report [2], an expression was derived for the cancellation ratio of scattered jamming in the case of strictly band-limited interference. The spectral band limiting can occur either at the interference source or in the receiver's preceding cancellation in the main and auxiliary channels. The noise spectrum was assumed rectangular, i.e., constant over a band of width $1/\Delta$ and zero outside of this region. Under this assumption, it was shown that the cancellation ratio is

$$\text{C.R.} = 1 - \sum_{n=-N}^{N} \frac{1}{2L} \int_{-(L+n)}^{(L-n)} \operatorname{sinc}^2(\pi u)\, du \,. \tag{4.1}$$

This cancellation ratio is the ratio of the power residue in the output of the tapped delay line canceler to the power in the main channel without cancellation.

This expression has been evaluated by numerical integration for a variety of cases, i.e., combinations of L and N. The number of delay taps in the auxiliary channel is $(2N + 1)$ in the above equation. The width of the region of uniformly distributed scatterers is 2 L, measured in relative delay time in tap spacings.

To obtain values for C.R., the following function was computed numerically as a function of K:

$$U(K) = \left[\int_0^K \mathrm{sinc}^2\left(\pi u\right) du\right] \mathrm{Sgn}(K) \ . \tag{4.2}$$

The numerical integration was performed with an accuracy of 8 decimal places.

Values for the cancellation ratio were obtained using

$$C.R. = 1 - \frac{1}{2L} \sum_{n=-N}^{N} \left[U\left(L + n\right) + U\left(L - n\right)\right] . \tag{4.3}$$

In all cases, the tap delay interval is centered at the middle of the scattering region.

Results for a variety of cases are shown in Table 1. Note that a large number of taps is required to obtain ~ 25 dB of cancellation when the scattering region is small. For example, with a scattering region extended over only 2 tap spacings (2 L), a canceler with 79 taps yields only 25.9 dB of cancellation. This large residue is due to the slow decrease in the sinc^2 function. When the delay line tap interval is roughly matched to the scatterer delay interval, e.g., 2 L = 14 and 2 N + 1 = 15, the cancellation ratio is less than 20 dB.

Table 1

DEPENDENCE OF CANCELLATION RATIO
ON NUMBER OF TAPS AND WIDTH OF SCATTERING REGION

[Cancellation ratios are in dB.]

Number of Taps (2N+1)	Width of Scattering Region (2L)						
	2	6	14	26	38	50	70
3	11.3	2.9	1.0	0.5	0.4	0.3	0.2
7	15.3	13.9	2.9	1.3	0.9	0.7	0.5
15	18.7	18.5	16.4	3.7	2.2	1.5	1.0
27	21.2	21.2	20.8	18.3	5.3	3.3	2.1
43	23.3	23.2	23.1	22.6	21.3	8.4	4.1
59	24.6	24.6	24.6	24.3	23.9	23.0	7.9
79	25.9	25.9	25.9	25.7	25.5	25.2	23.9

REFERENCES

1. Brennan, L. E., and I. S. Reed, *Automatic Cancellation of Scattered Interference*, 1st Quarterly Progress Report submitted to the Naval Air Systems Command by Adaptive Sensors, Inc., under Contract N00019-80-C-0570, March 1981.

2. Brennan, L. E., and I. S. Reed, *Noise Residue After Cancellation of Scattered Interference*, 2nd Quarterly Progress Report submitted to the Naval Air Systems Command by Adaptive Sensors, Inc., under Contract N00019-80-0570, June 1981.

3. Brennan, L. E., and I. S. Reed, *Cancellation of Scattered Radar Interference from a Moving Platform*, 3rd Quarterly Progress Report submitted to the Naval Air Systems Command by Adaptive Sensors, Inc., under Contract N00019-80-C-0570, September 1981.